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Experimental Study of the Mechanism of Skiing Turns. II. Measurement of Edging Angles

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In the previous paper, we showed that the edging angle of the ski is one of the basic factors contributing to skiing turns. In this paper, we conducted a quantitative evaluation of edging angles and measured the geometrical parameters of a sliding ski. We found that when a line perpendicular to the direction of movement of the ski within the bottom of the ski (on the plane of the trace) was horizontal, the ski made a straight descent, while when the line had a positive or negative angle from the horizontal plane, it showed a right or left turn, respectively.

KEYWORDS: ski, turn, edging angle, uphill turn, downhill turn

§1. Introduction

In the previous paper,[1)] we presented the results of our experiment which examined the turning motion of the ski in the laboratory by using a model ski and sand for snow. The experiment showed that the edging angle and the upward flection of the forebody of the ski were the primary factors contributing to the turn of the ski as it begins to turn from a straight downhill run to an uphill turn.

This paper also discusses another experiment in which we measured the edging angle of the ski in a turning motion, that is, the angle between the bottom surface (trace surface) of the ski and the sand surface. The equipment used was the same as that used in the previous experiment. Progressing from the rough estimate to the detailed evaluation, we found that the direction of the turning of the ski and the angle between the horizontal and the underside of the ski have a close relationship. Here we report the results of the quantitative evaluation of skiing dynamics.

§2. Experimental Methods

2.1 *Ski*

As shown in Fig. 1, our model ski was made of vinyl chloride ($19 \times 2 \times 0.08$ cm^3) with no camber or sidecut. The tip of the model ski was bent upward. The ski had an aluminum cylinder and an iron plate to permit the alteration of the center of gravity by moving magnets on the iron plate. The total weight including the magnets was 36 g. The ski was assigned a fixed coordinate system (x, y, z), the center of gravity being at (x_0, y_0, z_0). In our experiment, we changed only x_0, keeping y_0 and z_0 at 0.00 cm and 1.19 cm, respectively. We slid the ski on a surface of fine sand (slope angle: $\alpha = 26°$; sand grain diameter: 0.05 cm or less). The sand slope was contained in a box measuring $180 \times 80 \times 5$ cm^3 with a sand depth of 1 cm. The sand surface was given a coordinate system where the Y axis was in the direction of the fall line and the X axis was perpendicular to the Y axis. The motion of the ski was photographed at 0.25 s intervals.

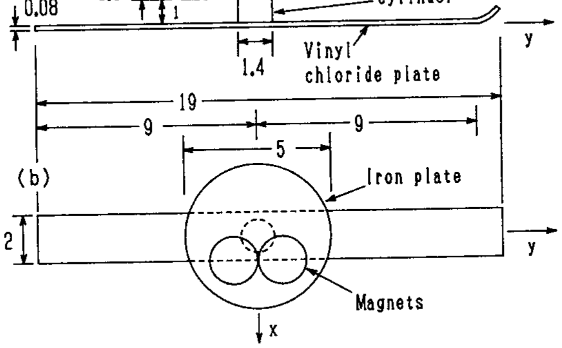


Fig. 1. Model ski made of vinyl chloride and its coordinates x, y, z: (a) side view; (b) top view.

2.2 *Measurement of L, δ, θ, β, β_0 and R*

We analyzed the locus L of the ski's center of gravity from the photographs. Figure 2(a) shows the method of measuring the angle δ between the fall line and the longitudinal direction of the ski, and the angle θ between the fall line and the tangent of the locus of the ski's center of gravity. Figure 2(b) shows the method used to measure the edging angle β between the line perpendicular to the sliding direction in the ski's bottom surface and the sand surface, that is, *the edging angle of the ski with respect to the sand plane*, by means of a triangle. In the measurements, a was approximately 10 cm, the measurement error for b was within ± 0.5 mm, and the error of measurement of β ($\Delta\beta$) was $\Delta|\beta| < 0.3°$. The radius of curvature R was obtained from the locus of the center of gravity. We defined R as positive when the locus was in a right-hand turn and as negative when the locus was in a left-hand turn.

From Fig. 3, angle β_0 between line CJ perpendicular to the sliding direction in the ski's tracing surface and the horizontal plane, that is, *the edging angle of the ski with respect to the horizontal plane*, is obtained by using α, β, and θ as follows:

if $\theta \neq 0$: $\mathrm{AC} = 1$, $\mathrm{AD} = \mathrm{s}$, $\mathrm{BD} = \mathrm{q}$, $\mathrm{CD} = \mathrm{p} = 1/\tan\theta$,
$\mathrm{AB} = \mathrm{u} = \tan\alpha$,

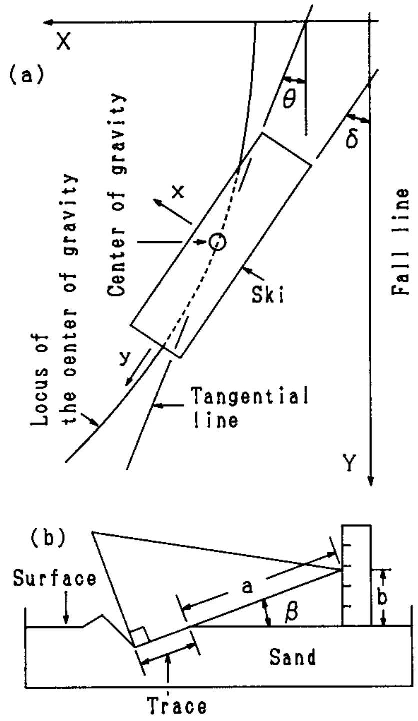


Fig. 2. (a) Angles δ and θ. δ is the angle between the direction of the ski and the fall line. θ is the angle between the tangent of the locus of the ski's center of gravity and the fall line. (b) Angle β between the ski's trace surface and the sand surface.

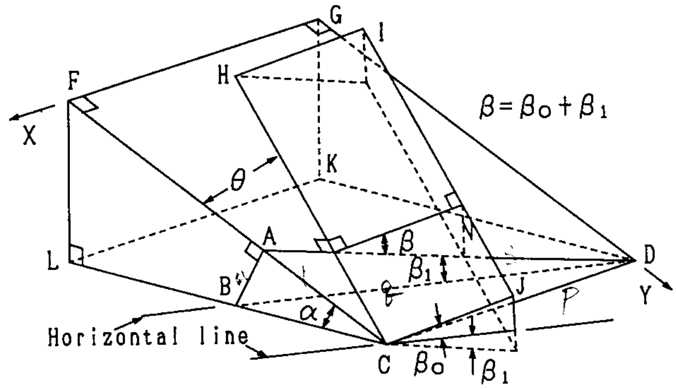


Fig. 3. Edging angle β with respect to the sand plane (sand surface) and edging angle β_0 with respect to the horizontal plane. β_0 is the angle between line CJ perpendicular to sliding direction CH of the ski's trace surface CJIH and the horizontal plane CDKL.

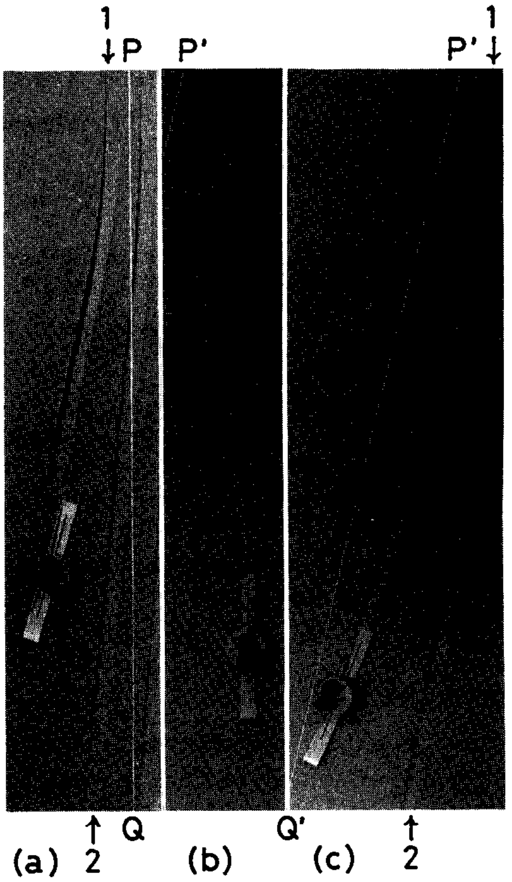
$$BC=v=1/\cos\alpha,\ s^2=1+p^2,\ q^2=v^2+p^2,$$

$$u^2=s^2+q^2-2sq\times\cos\beta_1,\ \beta_0=\beta-\beta_1;$$

if $\theta=0$: $\beta_1=0$, $\beta_0=\beta$.

The measurement error of angle β_0 ($\Delta\beta_0$) is, as described above for $\Delta\beta$, $\Delta|\beta_0|<0.3°$, where CDKL is the horizontal plane and CDGF is the sand surface (the sand plane). CF represents the fall line and CH a line (in the direction of motion, that is, sliding) parallel to the tangent of the locus of the ski's center of gravity. CH is generally different from the longitudinal direction of the ski. CJIH is a plane in which the underside of the ski is parallel with the tracing surface of the ski. L, δ, θ, β, β_0 and R are expressed using L through R.

Figure 4 illustrates the ski during a sliding motion.



(a) (b) (c)

Fig. 4. The ski in sliding motion with turns. The white line PQ is a thread stretched 5 cm above the sand surface in the direction of the fall line. P'Q' is a thread stretched at an angle of 12.5° from the fall line. (a) Uphill turn starting from a straight downhill run. (a)-1 Trace where $x_0=0.50$ cm, 3 s after start. (a)-2 Trace where $x_0=0.44$ cm. (b) &(c) Downhill and uphill turns starting from traverse. (b) Trace where $x_0=-0.19$ cm, 3 s after start. (c)-1 Trace where $x_0=0.50$ cm, 2.75 s after start. (c)-2 Trace where $x_0\fallingdotseq 0.44$ cm.

Figures 5 and 6 show pictures of the ski in motion taken at 0.25 s intervals.

2.3 *Definition of descent motion*

The definitions related to descent motion adopted in this paper are as follows.

(A) Motion with $\theta=$constant is called a straight descent. This motion is further classified into the following three types.

(A1) Case with $\theta=\delta$:

(1) if $\theta=0$, it is a straight downhill run, and (2) if $\theta\neq 0$, a traverse.

(A2) Case with $\theta\neq\delta$ and $\theta=0$:

(1) if $\delta=$constant, there is a straight descent (straight downhill run) with sideslipping, and (2) if $\delta\neq$constant, the motion is a straight downhill run with rotation about the center of gravity.

(A3) Case with $\theta\neq\delta$ and $\theta\neq 0$:

(1) if $\delta=$constant, the motion is a traversal with sideslipping, and (2) if $\delta\neq$constant, it is a traversal with rotation about the center of gravity.

(B) A motion with varying θ is called a turning descent. It is classified into the following two types.

(B1) Case with $\delta=$constant: turning with sideslipping.

(B2) Case with $\delta\neq$constant: turning with rotation

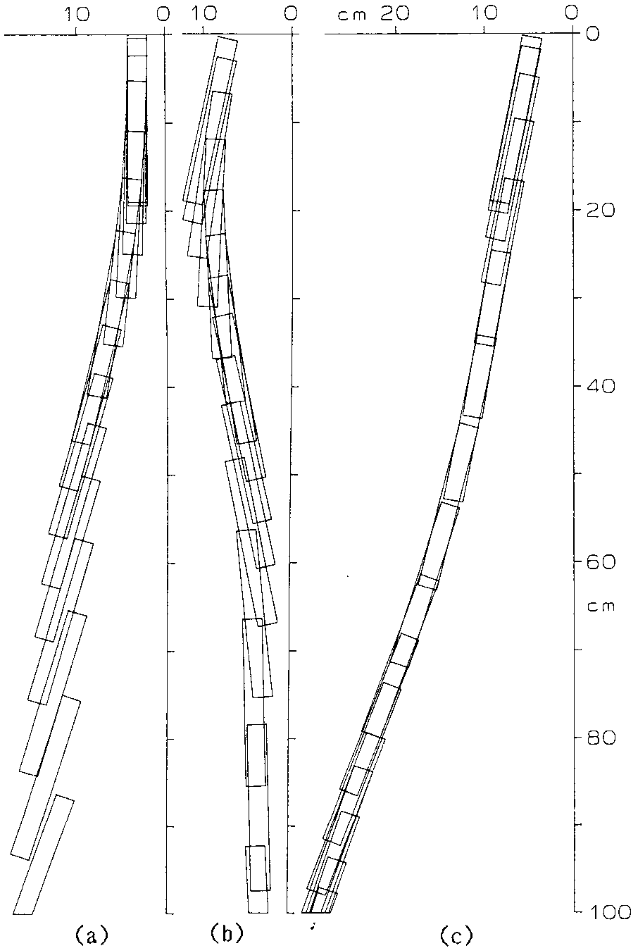


Fig. 5. Motion of Fig. 4 at intervals of 0.25 s. Figures 4(a)-1, 4(b) and 4(c)-1 correspond to Figs. 5(a), 5(b) and 5(c), respectively.

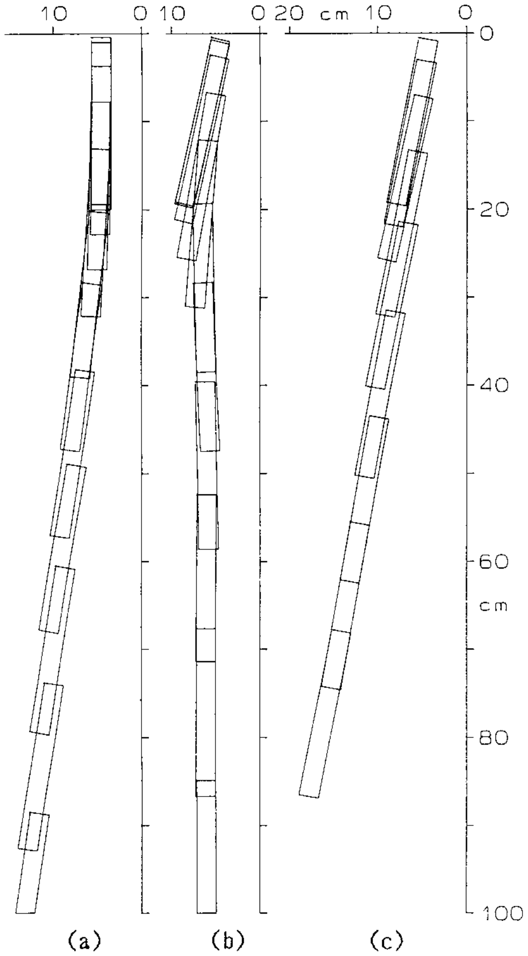


Fig. 6. (a) Motion of Fig. 8 at intervals of 0.25 s. (b) Motion of Fig. 10 at intervals of 0.25 s. (c) Motion of Fig. 11 at intervals of 0.25 s.

about the center of gravity.

(C) A turning descent with $\mathrm{d}|\theta|/\mathrm{d}Y>0$ is called an uphill turn, and that with $\mathrm{d}|\theta|/\mathrm{d}Y<0$ is called a downhill turn.

§3. Results

3.1 *Turning from straight downhill motion*

The ski was set to run initially parallel to the fall line. As is discussed below, when the center of gravity was $|x_0|<0.50$ cm, the ski slid with a downhill turn or straight descent after making an uphill turn. When the center of gravity was $|x_0|>0.50$ cm, the ski fell toward the slope after an uphill turn.

Figure 4(a)-1 gives a sample trace of the ski with $x_0=0.50$ cm. Figure 5(a) gives the motion of the ski at 0.25 s intervals. L through R are shown in Fig. 7. In Fig. 7, the Y axis is parallel to the fall line. In Fig. 7(a), the starting point of the ski is at $Y=9$ cm. At $Y=35$ cm, (1) θ reached a maximum and the turning direction of the locus changed; (2) before and after this, the radius R became $\pm R$ and the sign of R changed, as shown in (c); and (3) as shown in (b), β_0 became zero. However, there were no particular changes in δ or β. If we define Y_0 as the point where the values of θ, R and β_0 show a notable change, Y_0 becomes 35 cm. Y_0 is the inflection point (dotted line) at which the sign of the radius of curvature of the locus (c) changes. When $Y<35$ cm, $\beta_0>0$ and the ski turns uphill. When $Y>35$ cm, $\beta_0<0$ and the ski turns downhill. Moreover, when $Y>60$ cm, $\beta_0\approx 0$, $\theta\approx$ constant and $\theta\neq\delta$, and the ski traversed with rotation and sideslipping.

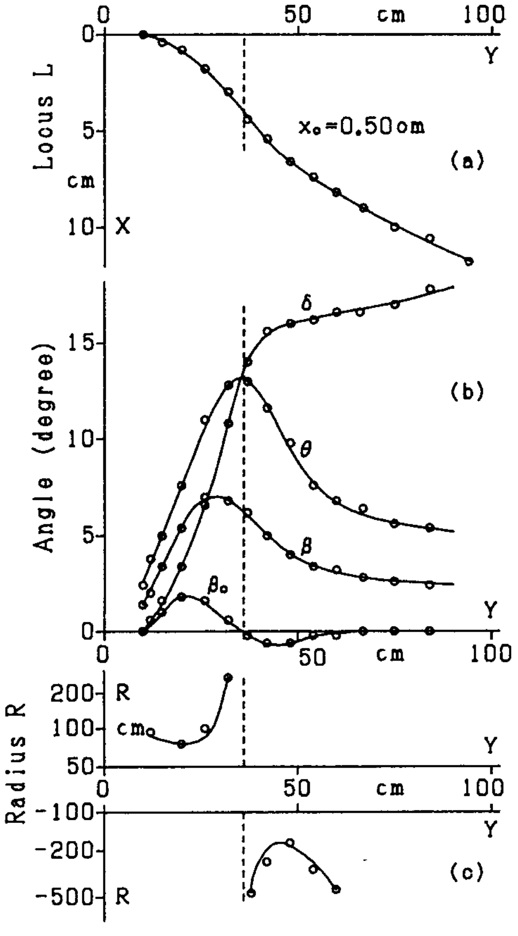


Fig. 7. L, δ, θ, β, β_0 and R of Fig. 4(a)-1.

Figure 4(a)-2 shows the trace of the ski at $x_0=0.44$ cm. Figure 6(a) shows its motion. Figure 8 shows L through R. Where Y was less than 35 cm, β_0 was positive. This results in an uphill turn. Where Y was greater than 35 cm, $\beta_0=0$, δ was constant, and θ was a constant not equal to δ. This gives traversal with sideslipping.

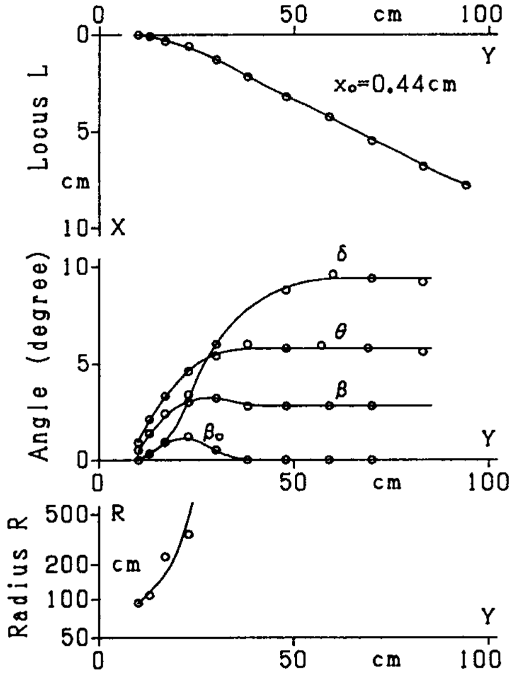


Fig. 8. L, δ, θ, β, β_0 and R of Fig. 4(a)-2.

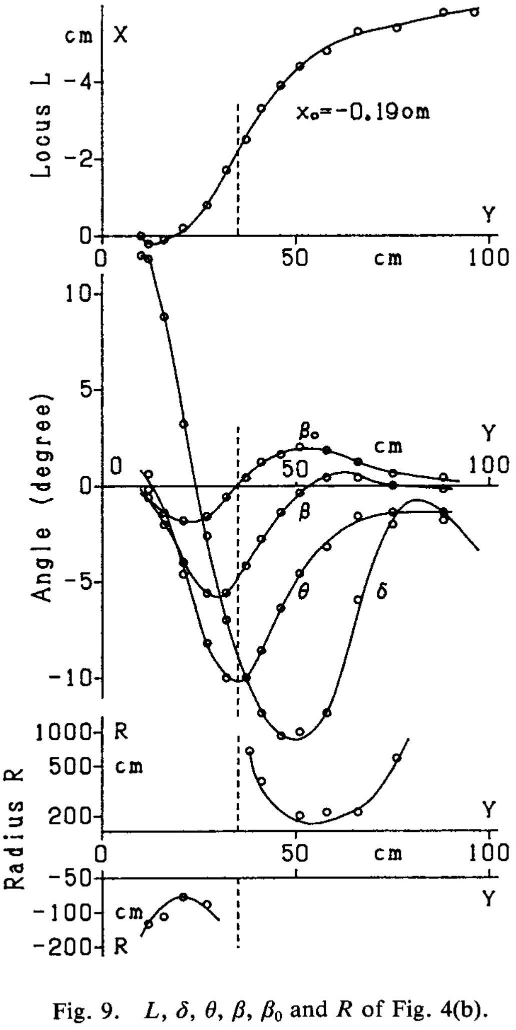


Fig. 9. L, δ, θ, β, β_0 and R of Fig. 4(b).

3.2 *Turning from traversal*

Sliding of the ski was started after the angle between the ski and the fall line was set to 12.5°. When the center of gravity was in the range of $-0.19\text{ cm}<x_0<0.50\text{ cm}$, the ski slid with uphill turns, downhill turns, sideslipping and/or traversing. Outside this range, the ski slid, then tumbled.

Figure 4(b) depicts the trace of the ski at $x_0=-0.19$ cm. Its motion is shown in Fig. 5(b). Figure 9 shows L through R. When Y was less than 14 cm, the ski exhibited a downhill turn. When Y was greater than 14 cm and less than 35 cm, the ski turned uphill. When Y was greater than 35 cm, the ski made a downhill turn. When Y was less than 35 cm, β_0 was negative. When Y was greater than 35 cm, β_0 was positive. Therefore, $Y_0=35$ cm.

Figure 6(b) shows the motion of the ski at $x_0=0.00$ cm. Figure 10 shows L through R. When Y was less than 15 cm, the ski made a downhill turn. When Y was greater than 15 cm and less than 25 cm, the ski turned uphill.

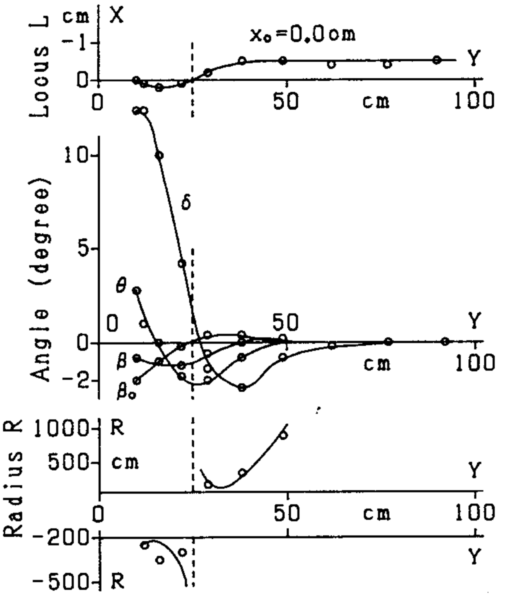


Fig. 10. L, δ, θ, β, β_0 and R where $x_0=0.00$ cm.

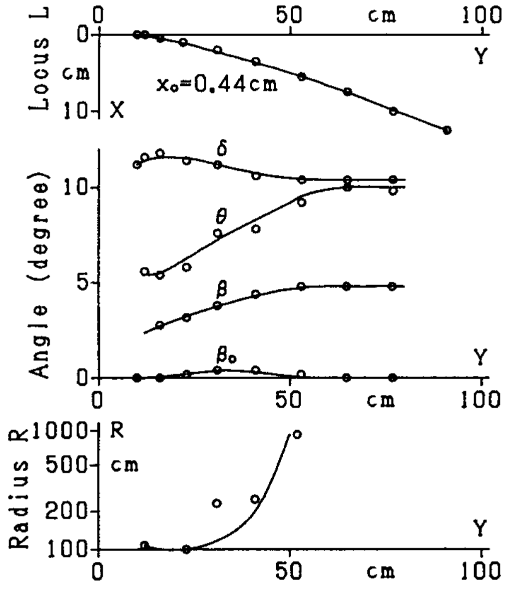


Fig. 11. L, δ, θ, β, β_0 and R of Fig. 4(c)-2.

When Y was greater than 25 cm and less than 60 cm, the ski made a downhill turn. When Y was greater than 60 cm, both θ and δ were zero, and the ski progressed in a straight downhill run. When Y was less than 25 cm, β_0 was negative. When Y was greater than 25 cm and less than 60 cm, β_0 was positive. When Y was greater than 60 cm, β_0 was zero. Therefore, $Y_0=25$ cm.

Figure 4(c)-2 depicts the trace of the ski at $x_0=0.44$ cm. Figure 6(c) shows its movements. Figure 11 shows L through R. When Y was less than 60 cm, β_0 was positive and the ski made an uphill turn. When Y was greater than 60 cm, β_0 was zero with $\theta=\delta=$constant. In this case, the ski underwent a traverse motion.

Figure 4(c)-1 shows the trace of the ski at $x_0=0.50$ cm. Figure 5(c) shows its movements. Figure 12 shows L through R. When Y was less than 45 cm, δ was about 10°. In this case, the ski made an uphill turn with sideslipping. When 45 cm$<Y<$75 cm, θ was approximately equal to δ. At this time, the ski showed a carving uphill turn.[2] When Y was greater than 75 cm, the ski made a downhill turn. When Y was less than 75 cm, β_0 was positive. When Y was greater than 75 cm, β_0 was negative. Therefore, $Y_0=75$ cm.

3.3 *Turns by a skier on snow*

Figure 13 depicts the trace of one-legged skiing by a skier. Figure 14 shows the measured values of L, θ, β_0, and R of the ski, indicating an uphill turn with β_0 at approximately 5° and $\alpha=15°$. In the measurements, the angle (slope angle) of the ground snow surface with respect to the horizontal plane was measured using an angular gauge and the fall line was then obtained. The position of the trace was determined based on these measurements. Thus angle β_0 of the trace plane with respect to the horizontal plane was measured using the above-mentioned angular gauges. Again, the error of these measurements is within 1°.

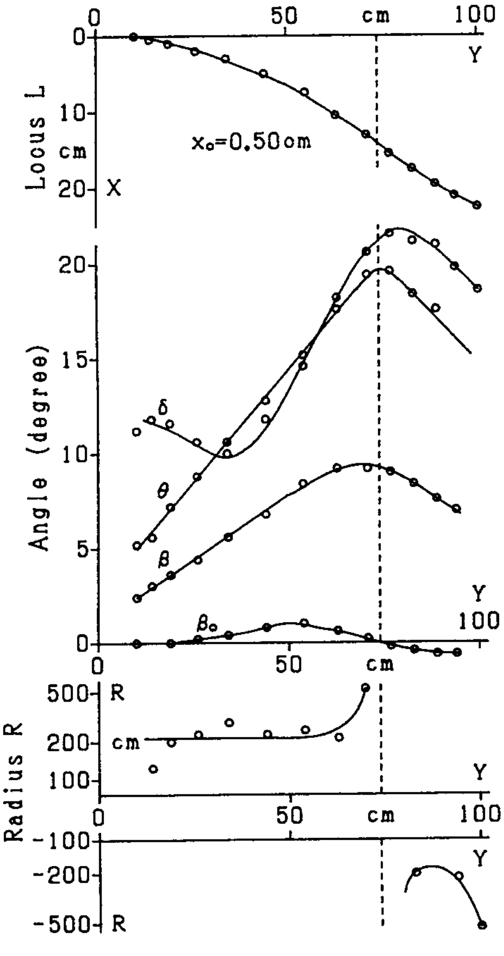


Fig. 12. L, δ, θ, β, β_0 and R of Fig. 4(c)-1.

Fig. 13. Trace of one-legged skiing.

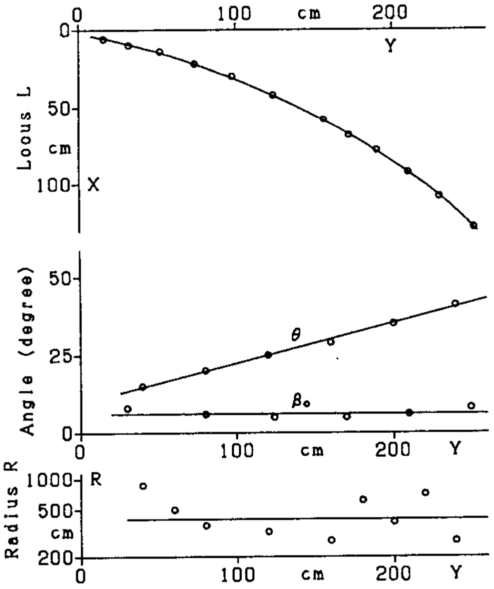


Fig. 14. L, θ, β_0 and R of Fig. 13.

§4. Discussion

4.1 *Straight descent*

We examined the relationship between straight descent and the edging angle β_0. Under the conditions (regions of locus) listed in Table I, every θ is constant and all the motions are straight descents. For these conditions, $\beta_0=0$. In condition (1) of the table, because $\theta\neq0$ and $\delta\neq$constant, the motion was traverse with rotation about the center of gravity. In condition (2), since $\theta\neq0$, $\theta\neq\delta$ and $\delta=$constant, the motion was traverse with sideslip results. In condition (3), since $\theta=\delta=0$, a straight