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Coefficient of Kinetic Friction of Snow Skis during Turning Descents

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On a snow plane, descents were performed by snow plows, stem turns, parallel turns and wedelns. The descents were photographed in sequence and these were used to draw the loci of the skis. Coefficients of kinetic friction between the skis and the snow during the turns were obtained from the loci; their values were between 0.01 and 0.3.

KEYWORDS: ski, turning descent, snow plow, stem turn, parallel turn, wedeln, coefficient of kinetic friction

1. Introduction

Kinosita *et al.*[1] measured coefficients of kinetic friction between skis and a snow plane during a straight descent. Shimbo[2] measured the coefficients of kinetic friction for model objects placed on a snow surface under various conditions. Shimbo reported that the effects of ski velocity and ski weight, including the skier (ski load), on the coefficient of kinetic friction are small when the ski material and snow temperature are constant.[2]

To analyze the motion of skis, the authors consider it necessary to measure the coefficients of kinetic friction between the skis and the snow during turning descents. To obtain coefficients of kinetic friction for the skis, one of the authors (Ichino) skied down a snow plane and performed snow plows, parallel turns, wedelns and two different types of stem turns by stemming out the uphill ski and the downhill ski. Photographs of the turning descents were taken in sequence and these were used to draw the loci of the skis.[3] Coefficients of kinetic friction during the turning descents were obtained from the loci. The coefficient of kinetic friction achieved its lowest value of about 0.01 just before the skis began to descend along a fall line while making a downhill turn. The coefficient attained its largest value of about 0.3 just as the skis began to make an uphill turn after descent along a fall line.

During skiing turns, rhythmic motions are essential. While descending by wedelns, periodic right and left turns are performed, resulting in periodic increases and decreases in the coefficients of kinetic friction. Skiers may feel this periodic increase and decrease in the coefficients as a rhythmic motion.

Gravity and centrifugal forces act on a skier during turning, and the apparent body weight of the skier increases. By obtaining the radius of curvature of the ski track and angular velocity from the ski track using the loci defined from the photographs, we estimated the apparent increase in body weight. The results showed a maximum of about a 1.1-fold increase in body weight. When turning on skies, a skier *feels* as if he/she *pushes* the skis in the direction of his/her feet. This *feeling* may arise from the apparent increase in body weight.

2. Experimental

2.1 Skiing conditions and photographs

The inclination of the ski slope, α, was 7°. The skier descended along several markers placed on the snow surface, and successive photographs of the skier were taken. Figure 1 shows an example of the photographs. Using these photographs and the method of Sahashi and Ichino,[3] the loci of the skis were drawn. Three photographs were taken every second. A 35 mm film camera with a 50 mm focal length was used. The skier (Ichino) is a ski instructor, certified by the Ski Association of Japan. The length of the skis was 180 cm. Hereafter, the trail left on the snow by a sliding ski is referred to as the track, and the location of a ski obtained from the analysis of photographs is referred to as the locus. The temperature of the snow was −6°C (tight powdered snow), and almost no wind was blowing.

Fig. 1. The skier during descent by snow plow. The photograph corresponds to ski number 6 in Fig. 3.

2.2 Tangential angle θ and radius curvature R of ski tracks and ski angle δ

Figure 2(a) shows a ski slope with an angle of inclination α. Line BC identifies the fall line (FL). The direction of descent of the skis is expressed as DB, and the angle formed between DB and BC is defined as the descent angle θ. The plane ABCD is redrawn in Fig. 2(b). The angle between the fall line and the ski is expressed as δ, and the center point of the 2 front corners and 2 rear corners of a ski is expressed as C. The front corner of the ski is taken as the corner in contact with the snow. The center points of the left and right skis are C_1 and C_2, respectively, and the mid-point of C_1 and C_2 is B. Since the skis move during a descent, the movements of the skis are periodically photographed and analyzed. The mid-point between left and right skis is represented as B as shown in Fig. 2(b). B_1, B_2 and B_3 shown in Fig. 2(c) are the mid-points observed in each photograph. Assuming a track passes through B_1, B_2 and B_3, we define the tangential angle θ at B_2 as the angle formed between the FL and the line connecting B_1 and B_3. The radius of curvature formed by the ski track is obtained from these three points. The tangential angle corre-

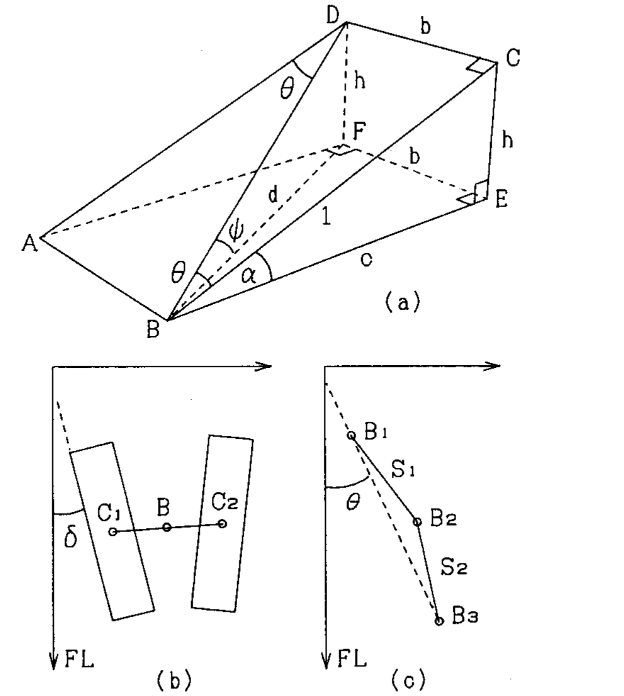

Fig. 2. (a) Idealized diagram of the slope, where α is the inclination of the slope and ψ is the apparent inclination of the slope. (b) Aerial view of the skis during descent on a slope. δ is the ski angle, and B is the center of both skis. (c) The ski track. B_1, B_2 and B_3 represent the movement of B. The direction of B_1–B_3, θ is a tangential angle of point B_2.

sponds to the descent angle mentioned above.

2.3 *Acceleration of the ski, a*

In Fig. 2(c), the distance between B_1 and B_2 is defined as S_1, and the mean velocity between B_1 and B_2 as v_1. The distance between B_2 and B_3 is defined as S_2, and the mean velocity between them as v_2. The time required to pass from B_1 to B_2 and B_2 to B_3 is the same, and is expressed as t. Acceleration due to gravity is g (=9.8 m/s^2), and the coefficient of kinetic friction between the ski and the snow surface is expressed as μ. If we define $S = (S_1 + S_2)/2$, $v_1 = S_1/t$, $v_2 = S_2/t$ and $2aS = v_1^2 - v_2^2$ when B_1, B_2 and B_3 are on the fall line, then a represents the acceleration of the skis at point B_2, and we obtain

$$a = g(\sin\alpha - \mu\cos\alpha) = (S_1 - S_2)/t^2.$$

2.4 *Apparent inclination angle of the ski slope, ψ*

As shown in Fig. 2(a), θ is the angle formed between DB and the fall line when the skis descend in the direction of DB, and the inclination of the slope at this time is defined as the apparent inclination angle of the slope ψ. Using CB=1, CE=DF=h, CD=EF=b, BE=c, BF=d, $h = \sin\alpha$, $b = \tan\theta$ and $c = \cos\alpha$, the apparent inclination angle ψ can be calculated as follows.

$$\psi = \tan^{-1}(h/d) = \tan^{-1}(h/\sqrt{b^2 + c^2}).$$

2.5 *Coefficient of kinetic friction during a turning descent, μ*

When B_1, B_2, and B_3 are not on a straight line, the ski tracks between B_1 and B_2 and between B_2 and B_3 are curved. If we assume that the lengths of these curves are expressed by lines S_1 and S_2, then the acceleration of the skis Gc at point B_2, which is on the curved track that passes through B_1, B_2, and B_3, can be expressed using S_1, S_2, t and ψ as follows.

$$Gc = g(\sin\psi - \mu\cos\psi) = (S_1 - S_2)/t^2. \quad (1)$$

The coefficient of kinetic friction μ between the skis and the snow during a turning descent is obtained as follows.

$$\mu = (\sin\psi - Gc/g)/\cos\psi$$

3. Results

3.1 *Snow plow*

Figure 3(a) shows the loci of a descent by a snow plow. An arrow shows the direction of the fall line (FL). The vertical axis in Fig. 3(b) represents the distance from the starting point on the fall line and the ski number. The angle formed between the fall line and the right ski, δ_R, is shown by an open circle, the angle of the left ski, δ_L, by a double circle and the tangential angle of descent, θ, by a closed circle. δ_R and δ_L are connected by a solid line and θ by a dotted line. In Fig. 3(c), the radius of curvature of the ski track R, the velocity of the skis v, the acceleration of the skis Gc, the coefficient of kinetic friction μ, and the apparent inclination of the snow plane ψ are shown. Points of inflection occur between right turns and left turns during descent, and radius of curvature R increases before and after the points of inflection. When $Gc > 0$, the velocity of the skis is accelerated, and when $Gc < 0$, the velocity of the skis is reduced. Points of local maxima and minima in Gc coincide with local minima and maxima in μ, respectively. Absolute values of Gc are less than g/10. When $\theta = 0°$, $\psi = \alpha = 7°$. The maximum value of the absolute value of θ is approximately 45°, and ψ becomes minimum at about 5°. We can see that when the velocity v of the skis increases, μ is small. Values of μ range from 0.05 to 0.2.

The opening angle between the skis of our skier is always about 20° during the snow plow. In Fig. 3, the tangential angle of the track, θ, is similar to the left-ski angle δ_L during left turns and right-ski angle δ_R during right turns. The left ski during left turns and the right ski during right turns are the inside skis. The body weight applied to an inside ski is reportedly less than that applied to an outside ski.[4)]

3.2 *Stem turn by stemming out the uphill ski*

As shown in Fig. 4, both skis are parallel and the angles of δ_R are nearly equal to those of δ_L before and after the point where the tangential angle $\theta = 0°$. Before θ becomes equal to 0°, Gc attains a local maximum and μ, a local minimum. The velocity of the skis begins to decrease when $\theta = 0°$ because δ becomes markedly different from θ, leading to an increase in the frictional force and an increase in the value of μ. The minimum value of μ is about half that of the snow plow. The maximum opening angle between the skis was about 10°, which was also half that of the snow plow. The velocity of the skis during a stem turn was higher than that of the snow plow, because the opening angle between the skis in the former was small. Values of μ ranged from 0.02 to 0.2.

3.3 *Stem turn by stemming out the downhill ski*

Figure 5 (stem turns by stemming out the downhill ski) is similar to Fig. 4 (stem turns by stemming out the uphill ski); there are few differences between the two cases. Values of μ ranged from 0.02 to 0.2.

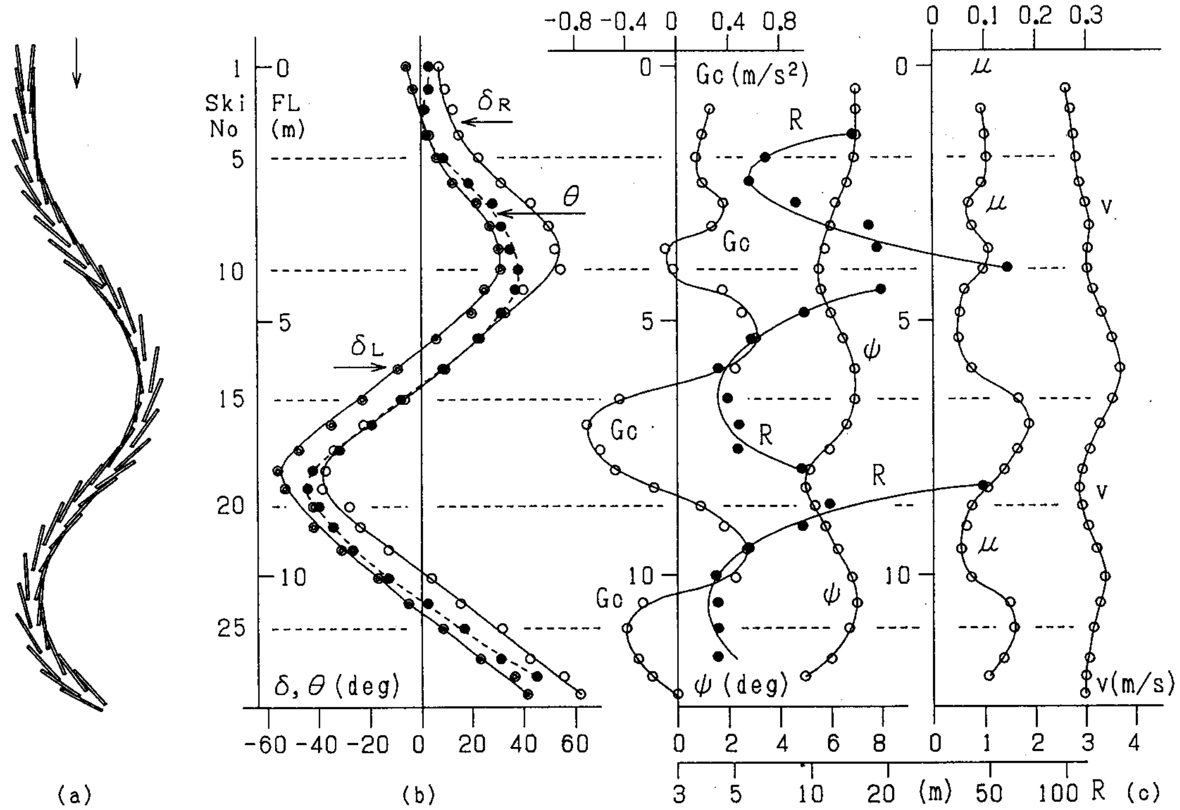

Fig. 3. (a) Locus of the snow plow. (b) Ski angle δ and tangential angle of the descent, θ. (c) Radius of curvature of the ski track R, velocity of the skis v, acceleration of the skis Gc, coefficient of kinetic friction μ and apparent inclination of the ski slope ψ.

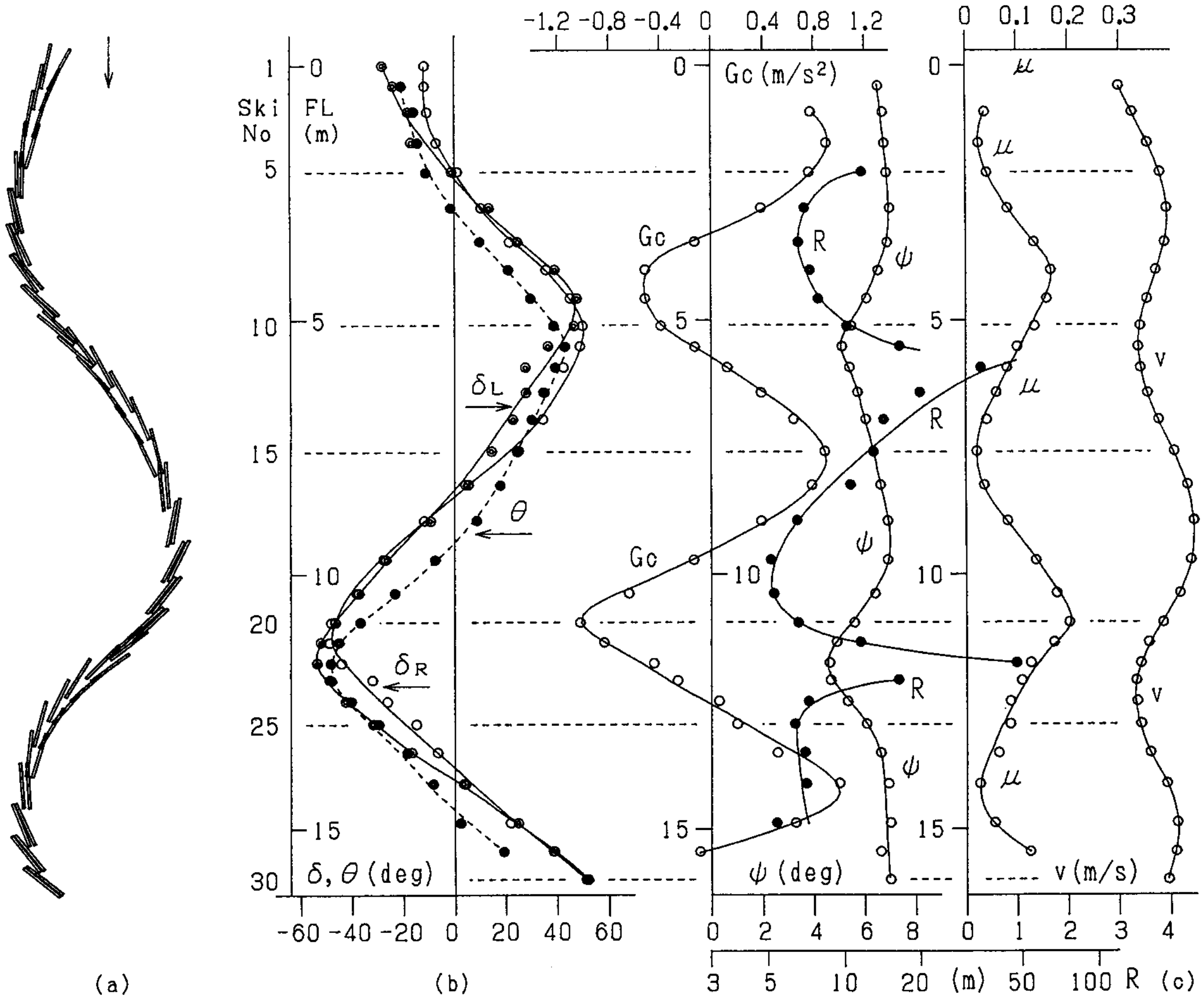

Fig. 4. Locus, δ, θ, R, v, Gc, μ and ψ of the stem turn by stemming out the uphill ski.

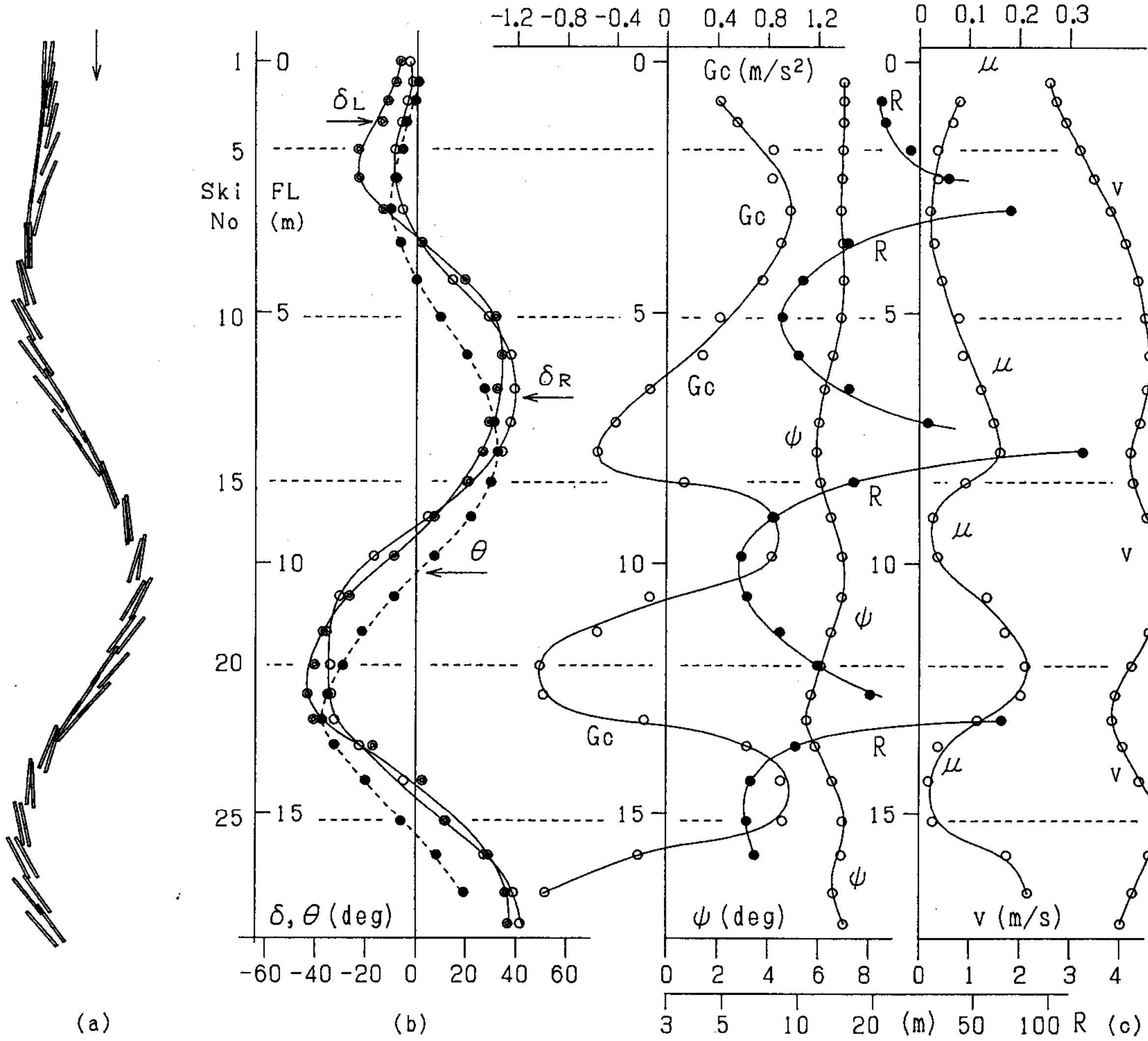

Fig. 5. Locus, δ, θ, R, v, Gc, μ and ψ of the stem turn by stemming out the downhill ski.

3.4 Parallel turn

As shown in Fig. 6, both skis are parallel to each other (δ_R nearly equals to δ_L) during a parallel turn. The velocity of the skis during parallel turns was higher than that during stem turns, probably because the opening angle between the skis during parallel turns was always nearly equal to zero. Values of μ ranged from 0.01 to 0.3.

3.5 Wedeln

As shown in Fig. 7, right and left turns alternate repeatedly during a wedeln. A local maximum in Gc and a local minimum in μ and vice versa are produced periodically. During a wedeln both skis are more truly parallel than those during parallel turns. From the four cases of descent examined so far, the absolute values of the tangential angle, θ, were less than 45°: however, in the case of a wedeln, the values were less than 20°. Because of this small variation in θ and the fact that the left and right skis are maintained in parallel, the velocity of the skis is high. Values of μ ranged from 0.01 to 0.3.

3.6 Centrifugal force and increase in body weight

We superimposed the locus of the skis during the parallel turn shown in Fig. 6(a) onto circles as shown in Fig. 8. As indicated, the loci of the skis formed arcs of circles with radii $r_1 = 7$, 6 and 5 m in this sequence. Hereafter, we will discuss the turns with respect to the circle with a 6 m radius. The angle of the arc on which the ski locus is overlapped is 1.24 rad. The angular velocity of the skis $\omega = 0.93$ rad/s. Here we assume the following: (1) The snow surface is a horizontal plane. (2) The dotted circle (radius: r_2) located inside the circle ($r_1 = 6$ m) is a projection of the skier's center of gravity to the snow surface. (3) Both the center of the skis and the center of gravity of the skier turn with the same angular velocity ω. If we focus on the region near point A in Fig. 8, which is in the middle of the overlapping parts of the circle and the ski locus, the three assumptions are reasonable. Figure 9(a) shows a diagram in which the skier at point A was observed from the front. G is the center of gravity of the skier. The center of gravity is inclined from the perpendicular by an angle ε. The distance between the skis and the center of gravity is h, and p represents the projected length of h on the snow. As shown in Fig. 9(b), the centrifugal force applied to the center of gravity is $Mr_2\omega^2$, and gravity is Mg. At a position near A in Fig. 8, the equation, $\tan\varepsilon = r_2\omega^2/g$, must hold. The following equation can be obtained from Fig. 9.

$$r_1 = r_2 + h\sin\varepsilon = r_2 + hr_2\omega^2/\sqrt{g^2 + (r_2\omega^2)^2}.$$

By assigning r_1, h, ω and g to the above equation, we obtain the value of r_2. The magnification factor due to the apparent increase in body weight, Mc, is

$$Mc = \sqrt{g^2 + (r_2\omega^2)^2}/g.$$

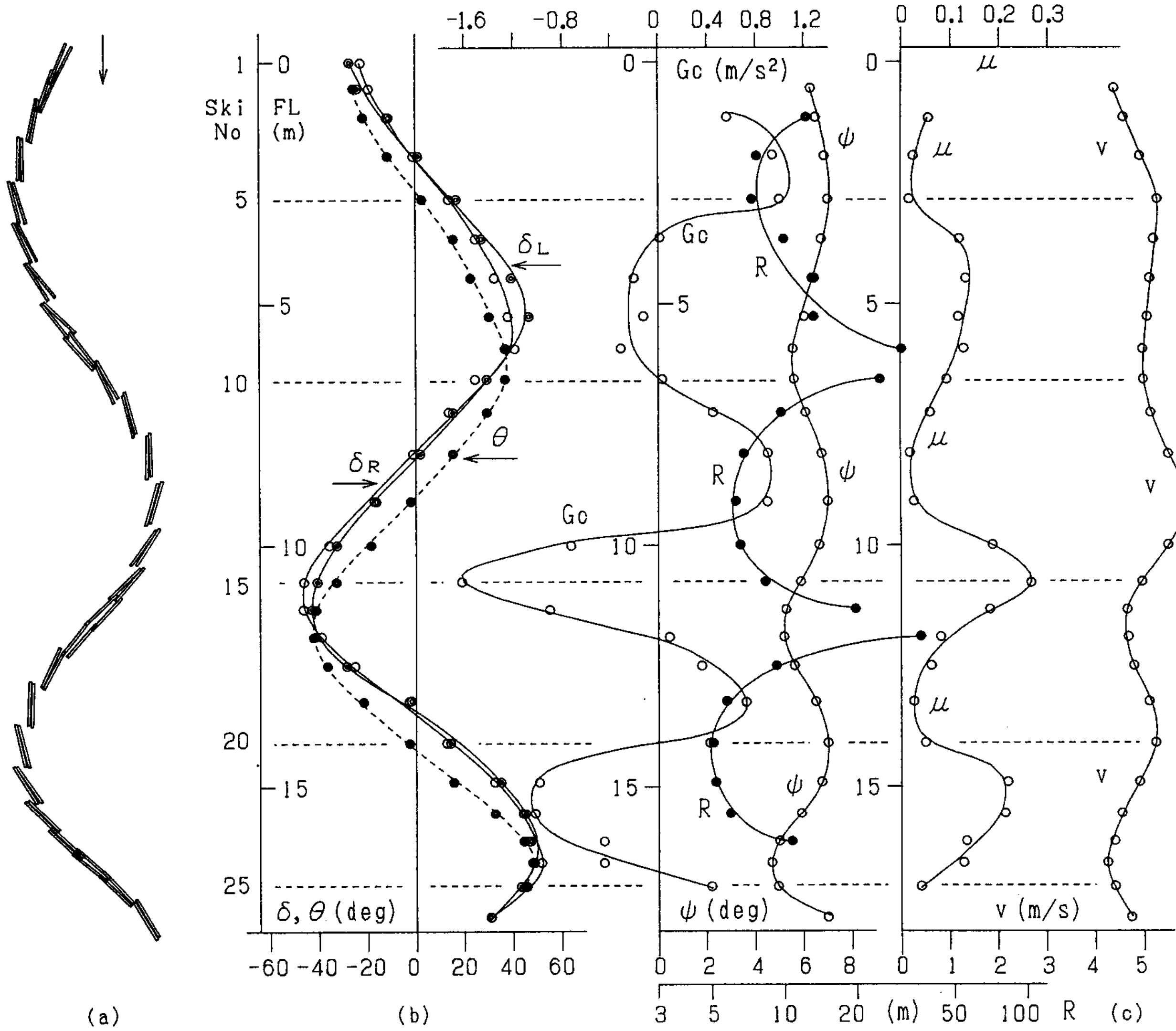

Fig. 6. Locus, δ, θ, R, v, Gc, μ and ψ of the parallel turn.

By assuming $h = 1$ m, we obtain the values of Mc listed in Table I. During the descent shown in Fig. 6, body weight increased by a maximum of about 1.1-fold due to the centrifugal force. When $r_1 = 6$ m, p and the magnification factor Mc varied with the change in h as shown in Fig. 10.

4. Discussion

4.1 *Snow plow*

The snow plow is a method of turning descent by a skier, in which both skis form a V shape as shown in Fig. 3(a). A similar type of loci are observed during a stem turn by stemming out the uphill ski as shown in Fig. 4(a) (ski numbers from 23 to 26). One of the characteristics of snow plows is that the inside ski shows almost no skidding, but the outside ski shows a large degree of skidding during a turn.

Evans *et al.*[5] drew an explanatory diagram of a turning descent using snow plows, and exhibit a large degree of skidding for an inside ski and no skidding for an outside ski during the turns. They do not describe how they achieved their drawing. Their result completely disagrees with our observational results. This disagreement may be derived from the difficulty of drawing ski turns.

4.2 *Parallel turn*

A parallel turn can mean either of the following. First, both skis are parallel to each other ($\delta_R \approx \delta_L \approx \delta$) during a descent by a parallel turn ($\delta \neq \theta$). Second, the skis descend in the direction of the parallel skis while turning ($\delta \approx \theta$). The latter means that a descent by parallel turn is similar to a descent by carving turns.

The photographs and drawings of the parallel turns exhibited in many skiing instruction books are placed in sequence so that the skis appear to descend in the direction of the parallel skis while turning.[4–6] This may be viewed as an accurate expression of parallel turns by the majority of skiers. According to the ski loci of stem turns and parallel turns shown in Figs. 4–6, several loci of skis are similar in shape before and after descents along the fall line. A difference of about 20° between δ and θ is observed in the skis in these positions as shown in the graphs. Thus, there seems to be a large difference between the perception of what occurs and the actual behavior of skiers.

4.3 *Rhythmic descent*

Skiing turns consist of left and right turns. According to our analysis of turns, we can see that when a skier begins an uphill turn after descending along the fall line, the value of μ achieves a local maximum. Based on our experience, when the period between a left turn and a right turn becomes small and equal in length, the turning descent has a rhythmic pattern. The position at which μ becomes a local maximum seems to be the point where this rhythmic pattern is produced.