The Course of our ski research
2. Edging angle of the ski with respect to the horizontal plane

スキー研究の軌跡
2．水平面への角付け角

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Summary
When the edging angle β_0 of the ski with respect to the horizontal plane is defined, the turning direction is governed by the sign (positive/negative) of the edging angle β_0. The edging angle β of the ski with respect to the inclined plane of a ski slope does not influence the turning direction, contrary to conventional belief. This finding is explained schematically.

キーワード：スキー，スキー滑降，回転機構，角付け角
Key words: ski, ski sliding, turning mechanism, edging angle

1. Introduction

We herein summarize what we presented in the Bulletin of Daido Institute of Technology in 2000.[1)]

(1) The major types of ski sliding include straight descents and turning descents. Aside from the level of the technique, straight descents are not very difficult to perform. However, turning descents are difficult and cannot be performed easily. Accordingly, the technique for skiing means the "technique for turning descent."

(2) According to skiing textbooks of all eras, a ski technique common to turning descents is "twisting the body of the skier toward the direction of the turn to turn the skis."

(3) We discovered the following based on our experiment: when β_0 is defined as the edging angle of the ski with respect to the horizontal plane, $\beta_0 \neq 0°$ corresponds to a turning descent and $\beta_0 = 0°$ to a straight descent.

(4) We define "a skier's feeling that he makes his skis turn" or "a skier's belief that he makes his skis turn" as "the skier's sense of making skis turn." Based on the experiments performed in a car, it was revealed that the skier's sense of making skis turn is "a sensory illusion."

We have proposed two ideas regarding the edging angle of skis.[1)] One is the "edging angle β_0 with respect to the horizontal plane." The other is the "edging angle β with respect to the inclined plane." The latter β has conventionally been called the edging angle, for example, as in textbooks on skiing.[2,3)]

In this paper, regarding the edging angle, we explain why β_0 is the factor that determines the turning direction, and why β is not, using diagrams.

2. Ski sliding

A skier on his skis stands on a snow plane. Since snow is powder, the accumulated snow changes its external shape due to the skier's weight. When a snow plane newly shaped by a skier is horizontal, the skis maintain static conditions. When the plane is inclined to the front/back or right/left direction, the skis slide in that direction. Let us call this *the principle of ski sliding*. This sliding may occur along a straight line or a curved line. Curved-line sliding corresponds to a turning descent.

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Fig. 1. Ski tracks formed on complexly deformed snow.

When sliding is straight, a track of sliding called a spur (track) is formed behind the sliding skis, from which the deformation of the snow plane due to the weight of the skis can be observed (Fig. 1).

3. Edging angle of the ski with respect to the horizontal plane

With the increasing number of skiers, the number of people discussing mechanisms of ski turns is also increasing. In addition, many textbooks on the subject have been published.[4,5,6] Our ski research has been prompted by these trends. Our first discovery was that by defining the edging angle β_0 of the ski with respect to the horizontal plane, $\beta_0 \neq 0°$ was satisfied during a ski turn,[7,8] as explained in Fig. 2.

Figure 2(a) shows a diagram of a ski slope. The inclination angle α of the slope is 30°, and the skier slides along the dotted line SOT on one foot. The area KHIJ is an enlargement of part of the sliding track of the ski passing through point O. The direction of sliding at point O is the same as the direction of the fall line FL. The angle β formed between the sliding plane and the inclined plane is 20°. This is the "edging angle β with respect to the inclined plane" which is the conventional definition of the edging angle.[1] An uphill turn made starting from point O in Fig. 2(a) while maintaining β=20° is schematically

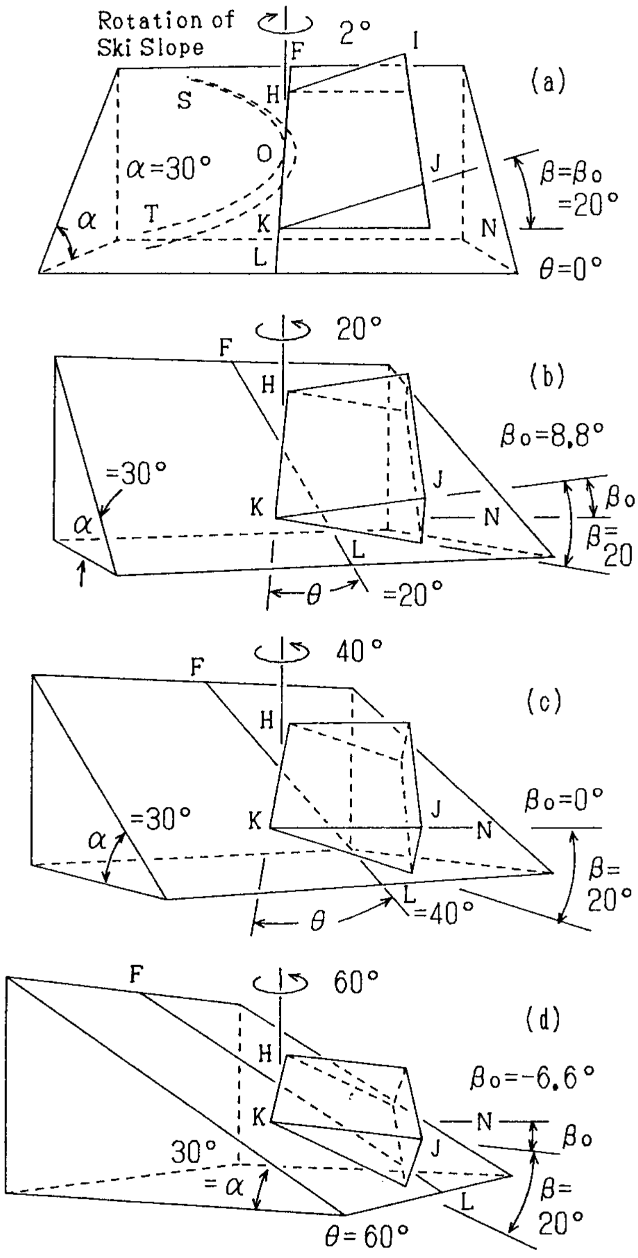

Fig. 2. Inclined planes of the ski slope and sliding planes of the ski.
(a) A skier performs one-legged sliding from the upper part of the inclined plane to SOT. The plane KHIJ is the enlargement of the plane of the sliding track at point O. The edging angle β of the ski is the angle formed between the sliding (track) plane and the ski slope, and FL is the fall line.
(b) When the skier makes an uphill turn and faces the direction 20°from FL, the angle formed between the sliding plane JK and the horizontal plane NK is the edging angle β_0.
(c) The skier faces the direction 40° from FL during a straight descent.
(d) Downhill turn.

illustrated in Fig. 2(b). The angle formed between the sliding direction of the ski and FL is θ=20°. Figure 2(b) is a schematic of the ski slope rotated by 20° to the left, in order to present a figure in which the ski is observed from the sliding direction. The angle θ is the tangential angle of the curved track. The line NK normal to the sliding direction is on the horizontal plane. The angle formed between the lines NK and JK is β_0; this is "the edging angle β_0 with respect to the horizontal plane ." In Fig. 2(b), $\beta_0=8.8°$. The sliding plane of the ski has two directions of inclination: the HK direction and the JK direction. Based on *the principle of ski sliding* noted in Sec. 2, it can be understood that the ski continues to turn uphill on this sliding plane. When the ski reaches the position shown in Fig. 2(c) by turning descent, the angle formed between lines NK and JK becomes zero.

Under this condition, the ski may slide in the direction of HK, but not in the direction of JK since the direction of JK is on the horizontal plane. In other words, the ski descends straight in the direction of HK, making $\beta_0=0°$. Figure 2(c) shows what is generally called by skiers a conventional traverse. We assume that the ski turned uphill and was in the state shown in Fig. 2(d).

Here, $\beta_0=-6.6°$. Under the condition shown in Fig. 2(d), the plane of ski descent inclines downward in the directions of HK and KJ. Here, we assumed an uphill turn, but we can see that this is incorrect based on *the principle of ski sliding*; the turn must be a downhill turn. In Fig. 2(d), $\beta_0<0°$ holds, whereas the edging angle β satisfies $\beta=20°$. Thus, it can be understood that the direction of turning descent is not determined by the edging angle β. The factor which governs ski turns is "the edging angle β_0 with respect to the horizontal plane." $\beta_0>0°$ holds for uphill turns and $\beta_0<0°$ holds for downhill turns. These results satisfy *the principle of ski sliding.*

The fact that the sliding plane of the ski has two inclinations (i.e., in the directions of HK and JK) explains the revolutional motion of the ski, but it does not explain the rotational motion of the ski. For rotational motions[9], other elements must be taken into consideration, as will be presented in our next article.

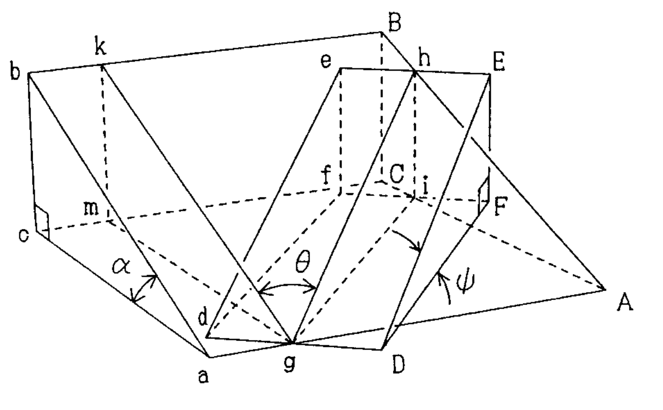

Fig. 3. The inclined plane aABb of the ski slope with inclination angle α.
The inclined plane dDEe of the ski slope with inclination angle ϕ. The line hg satisfies both the straight downhill run and traverse conditions.

In this paper, the turning descents are explained only in terms of revolution while neglecting rotation. This is also the case for Figs. 2(a) and 2(d).

Ikegami et al.[10] examined ski tracks, the center of gravity of a skier, edging angles and drift angles by analyzing ski turns using the DLT method, and discussed turning descents. The edging angle they measured is "the edging angle with respect to the inclined plane," which corresponds to β in our study. The inclination angle of the ski slope was $\alpha=13.5°$. If $\alpha\to0°$, then $\beta\to\beta_0$ and their theory becomes equivalent to our theory.

4. $\beta_0=0°$ holds for straight descent

If $\beta_0=0°$ holds for straight descent, then a straight downhill run and a traverse should be the same phenomenon. We explained this in the Bulletin of Daido Institute of Technology in 2000.[1] Let us briefly review it here. In Fig. 3, the plane aABb is an inclined plane of a ski slope with an inclination angle α. The plane dDEe is the inclined plane of the ski slope with an inclination angle ϕ. The respective bottom planes of these inclined planes, aACc and dDFf, are on the same horizontal plane. The line kg indicates a straight downhill run with an inclination angle α. The line hg indicates a straight downhill run with an inclination angle ϕ; at the same time, this line hg indicates a traversal line with an inclination angle α. If we assume that the width eE of the ski slope is 10 cm, the same as the width of the ski, then it can be easily understood that the inclined plane dDEe is the remaining track of the traverse. Both the straight downhill run and the traverse satisfy $\beta_0=0°$.

5. Deformation of snow plane

In many cases, ski sliding is performed on a deformed snow plane (track). Even on a prepacked solid snow plane, deformed tracks remain (Fig. 4).

Fig. 4. A ski track of a traverse remaining on a snow plane.

Fig. 5. The author (Sahashi) measuring the angle β_0 on a plane of a sliding track.

Fig. 6. One-legged sliding on a snow plane.

A snow plane is deformed according to *the principle of ski sliding*. The relationship between a ski turn and β_0 (the β_0 rule [8]) was discovered using a sand plane and a model ski,[7] and was confirmed on a snow plane with actual skis. This β_0 was measured on tracks formed by ski sliding (Fig. 5), which demonstrated that a track with $\beta_0 \neq 0°$ was formed on a plane with the tracks of ski turns. β_0 values described in Sec. 3 were also explained in terms of the deformation of the snow plane due to ski motions. One of the factors influencing ski turns may be the deformation of the snow plane. Materials which deform due to the weight of skis can be used instead of snow, on which a turning descent similar to that of skis on snow (snow skis) is possible.

Turning descents of skis can also be performed with one ski (Fig. 6).[8] To discuss the turning mechanisms of skis, it will be necessary to first discuss turning descents with one ski. In Sec. 3, explanations are given for one ski. We consider that ski turns made on two skis are the composite of ski turns made by one ski.

Shimizu,[11] Ohara[12] and Sakata[13] performed experiments of ski sliding using a ski robot, which performed edging on a carpet, to discuss the turning mechanisms of skis. The turning of carpet skis is not based on the deformation of sliding planes. We consider that *the principle of ski sliding* cannot be applied to sliding on a carpet. The β_0 rule for snow skiing does not hold for carpet skiing. To date, no experiments quantitatively showing the same principle between carpet skiing and snow skiing have been performed. Therefore, we consider that carpet skiing is based on a principle different from that for conventional snow skiing.

6. Research on skis and skiers

The subject of our study is "ski sliding," in which a skier wearing skis slides on snow at a typical skiing speed (approximately 10 m/s or less). During ski sliding, the friction between the skis and snow is significantly greater than that between the skier/skis and surrounding air. This is because the skis deform the snow plane and form a sliding track, generating much friction. Therefore, we consider that the appropriate sequence of analysis of the motion of the skis and skier is as follows: (1) How the skis move on a snow plane; (2) how the skier's stable posture is formed on his skis; and (3) how the skier changes his posture to change the edging angle, in order to change the magnitude and direction of the turn. The details are as follows.

(1) Since the friction between the skis and snow is large, most of the ski motion is governed by this friction. Accordingly, it is most important to investigate ski motions on a snow plane.[14] A two-dimensional analysis is appropriate for such an analysis.[15]

(2) For the skier to stand on his skis in a stable manner, we need to know the acceleration applied to the skier in the front/back and left/right directions. This can be determined from the changes in the positions of the skis.[14] When the change in the position of the center of gravity of the skis in the vertical direction (normal to the ski slope) is small, the above-mentioned two-dimensional analysis on a ski slope is sufficient. However, when the vertical change is large, three-dimensional analysis is required, which leads to complex analytical processes.

Item (3) above is extremely complex. Various operations of skis for each turning descent are described in the textbook entitled Ski Association of Japan.[16] Roughly speaking, if a skier can perform these operations, stable changes in the edging angles of skis can be achieved. Our research is still in stage (1), and only a part of (2) has been elucidated.

7. Conclusions

Skis descend straight when $\beta_0 = 0°$, and turn when $\beta_0 \neq 0°$. If a skier can stand on sliding skis in a stable manner, then a ski descent is achieved. However, it is